Characterizing neural dependencies with Poisson copula models

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The activities of individual neurons in cortex and many other areas of the brain are often well described by Poisson distributions. Unfortunately, there is no simple joint Poisson distribution that can incorporate statistical dependencies (i.e., correlations) between neurons. For this reason, neural population coding models often either assume that the individual neurons are independent, or transform the joint activity mathematically and model it using a multivariate distribution that naturally encodes dependency, such as the multivariate Gaussian. However, these solutions are sometimes poorly suited to describing neural population responses, failing to match either the marginal distributions of individual neurons or the detailed form of their dependencies. Here we develop a joint model for neural population responses using copulas, which allow Poisson marginal distributions to be combined into a joint distribution that captures dependencies between multiple neurons.

Copulas are mathematical objects that specify a joint distribution's dependency structure separately from its marginal structure [1]. More formally, copulas are joint probability distributions defined on the unit cube $[0, 1]^N$. Copula models are constructed by projecting the original variables through their cumulative density functions onto the unit cube. This is a central result for copula models and is formalized in Sklar's theorem [2], which states that every multivariate probability density $p(x_1, \ldots, x_N)$ can be written as the product of a copula density and marginal densities: $p(x_1, \ldots, x_N) = c(u_1, \ldots, u_N) \prod_{i=1}^{N} p(x_i)$, where $u_i = \mathrm{cdf}_i(x_i)$ and $c(u_1, \ldots, u_N)$ is a copula density. Copulas also provide a principled way to quantify non-linear dependencies that go beyond correlation coefficients (which are only appropriate for elliptical distributions), in a manner that is independent of rescaling of individual variables [1], and are applicable to the problem of estimating the mutual information between stimulus and response, as discussed in [3].

Here we present preliminary results on constructing a multivariate joint distribution for neural activity by choosing the marginals to be Poisson distributed, selecting an appropriate parametric family of copulas, and fitting the model parameters (of both the marginals and the copula) using Maximum Likelihood estimation. Different copula families are able to capture dependencies of different kinds (e.g., dependencies limited to the lower or upper tails of the distribution, or negative dependencies). The selection of an appropriate parametric family for the copula distribution can be addressed by cross-validation, and is the focus of current research.

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References

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